Lecture Notes

In Search of a Definition of Liquidity

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Abstract

What makes an asset liquid? Economists have long struggled to pin down this concept. These notes develop a search-theoretic framework that yields a precise, endogenous measure of liquidity: the expected time to sell an asset. We derive comparative statics on how time preferences and market thickness shape liquidity, and connect the formal results to classic definitions by Keynes, Hirshleifer, and others.

1 In search of a definition of liquidity

Learning targets. After studying these notes, you should be able to:

1. Explain why liquidity is difficult to define, and what makes a definition useful.
2. Formulate liquidity as an optimal stopping problem and derive the reservation price.
3. Perform comparative statics on how time preferences and market thickness affect liquidity.
4. Compute the expected time to sell an asset in a stationary search environment.

The definition of insanity is doing the same thing over and over again and expecting a different result.

This statement is often attributed to Albert Einstein. There is no evidence Einstein ever wrote or spoke it.[1] Further, insanity is a state of being seriously mentally ill. The quote describes a possible symptom of insanity but does not reach the qualification of a definition.

[1] See page 474 of "The Ultimate Quotable Einstein" (2010) by Alice Calaprice.

This section is (also) about making a good definition. A useful definition must be precise enough to distinguish instances, measurable enough to rank them, and grounded in a framework that explains *why* the concept matters. The epigraph above illustrates the pitfall: calling something a definition does not make it one. We will see that liquidity suffers from exactly this problem—and then we will try to fix it.

1.1 What is liquidity?

Liquidity is one of the most frequently invoked concepts in economics, yet one of the hardest to define. We follow an exposition by Lippman and McCall, 1986 that tries to provide a working definition. But first, consider what economists have said about a concept so close to their discipline.

1.1.1 Economic theorists past

Kenneth Boulding provides a broad definition but concedes that liquidity defies measurement.

> "Liquidity is a quality of assets which… is not a very clear or easily measurable concept." Boulding [1955, p. 310]

Boulding's concern is telling. A concept is an abstract idea generalized from particular instances. Height and physical distance are concepts with clear measurability: they invoke the notion of physical distance immediately, and we can rank any two instances. Weight is equally clear. Readability, however, is more difficult to measure as it depends on the reader: an expert finds informational introductions clunky and therefore unreadable while a novice requires them. For liquidity, we lack a universal metric, and without one, we cannot rank the liquidity of different goods.

John Maynard Keynes provides the following interpretation.

> "There is, clearly, no absolute standard of 'liquidity' but merely a scale of liquidity—a varying premium of which account has to be taken… in estimating the comparative attractions of holding different forms of wealth. The conception of what contributes to 'liquidity' is a partly vague one, changing from time to time and depending on social practice and institutions." Keynes [1936, p. 240]

Keynes agrees with Boulding that liquidity lacks a universal standard, yet he points to a scale or premium along which the liquidity of different forms of wealth varies. He also highlights that social norms could support the concept of liquidity.

Helen Makower and Jacob Marschak almost resign.

> "…'liquidity' has so often been used to cover all properties of money indiscriminately that it seems better not to use it for any of the separate properties of money. We thus resign ourselves to giving up 'liquidity' as a measurable concept: it is, like the price level, a bundle of measurable properties." Makower and Marschak [1938, p. 284]

On the same page, they note that "the fact that money is easily transformable (on the market) into other assets and is thus an effective instrument for maneuvering."

Jack Hirshleifer gives us a definition we can work with.

> "An asset's capability over time of being realized in the form of funds available for immediate consumption or reinvestment—proximately in the form of money." Hirshleifer [1968, p. 1]

1.1.2 Economic practitioners present

A definition proposed by Google (as good a source as any) is the **availability of liquid assets to a market or company**. Some stock traders might respond that liquidity is the **bid-ask spread in order books**, making **money the most liquid asset** as it resembles a mere exchange of one for one. Others point to **volume traded** as an indicator for how long conversion to money might take. For large funds it matters not only whether large orders can be filled but also how the market price reacts. A large sell order, for instance, can lower the price as the demand side becomes saturated. So this third tribe of stock traders might have the **price elasticity** in mind.

1.1.3 Economic expectations future

Our approach builds on the definitions of Hirshleifer and practitioners: liquidity describes the time to sell. We employ a rigorous economic framework, and search theory turns out to be the right tool.

An asset's liquidity is determined by

1. the difficulty of finding a buyer,
2. the time it takes to verify legal ownership and transferability,
3. the costs of holding the asset,
4. the impatience of the asset owner,
5. the sales price, and
6. the quantity transferred.

The last two points are familiar objects. Economists grant them special status: price and quantity are determined endogenously. We proceed with three simplifications. First, we limit our analysis to **one unit**. Second, we determine the liquidity of an asset **endogenously**, much like price and quantity. Third, we assume liquidity is determined by **rational** agents—after all, you can sell anything instantly if you drop the price to zero, while you can also outprice an asset from ever being sold.

Search theory is particularly helpful when we want to capture the difficulty of the transfer reflected in the first two points.

1.2 An economic framework

1.2.1 The environment

Time is discrete, runs forever, and starts at the end of period $t = 0$. A risk-neutral agent holds one asset and discounts future payoffs with $\beta = \frac{1}{1+\rho}$. Every round the agent pays a holding cost c_t, then receives an offer yielding x_t units of linear utility, where x_t is drawn from a distribution with cumulative distribution function F_t. The agent either accepts or declines. The game ends upon acceptance.

1.2.2 The optimal stopping rule

Say the agent accepts the offer in round τ. The payoff is

$$R(\tau) = \beta^{\tau} y_{\tau} - \sum_{t=1}^{\tau} \beta^{t} c_t$$

where y_τ depends on whether the agent can invoke any offer obtained until (and including) period τ, or just the latest offer:

$$y_\tau = \begin{cases} \max\{x_1, x_2, \ldots, x_\tau\} & \text{with recall} \\ x_\tau & \text{without recall} \end{cases}$$

For tractability, economic models usually do not allow recall.

The optimal round to sell is

$$\tau^* = \arg\max_{\tau} \mathbb{E}[R(\tau)] \tag{1}$$

and **liquidity is measured by**

$$\mathbb{E}[\tau^*]$$

the expected time until the asset is sold. But (1) is ill-defined: the decision is not fixed at $t = 0$ but can be revised over time.

1.2.3 Stationary solution

We resolve the ill-defined problem by imposing stationarity. Set $c_t = c$ and $F_t = F$ so that costs are constant and offers are drawn from a fixed distribution. Every period then repeats itself, and the stopping problem converts into a question of which offers to accept. The value of having a particular offer x and deciding whether to accept or decline it is

$$v(x) = \max\{x, \beta(-c + v)\} \tag{2}$$

where

$$v = \int v(x)\, dF(x) \tag{3}$$

is the expected value just before an offer x is drawn.[2]

The solution takes the following form: accept any offer above a threshold $\bar{x}$, the **reservation price**. The recursive, stationary nature of the problem ensures that the **continuation value**

$$\beta(-c + v)$$

is independent of time and the current offer. It is a constant equal to, say, $\bar{x}$. We accept the offer whenever it exceeds $\bar{x} = \beta(-c + v)$.

Figure 1 illustrates equation (2).

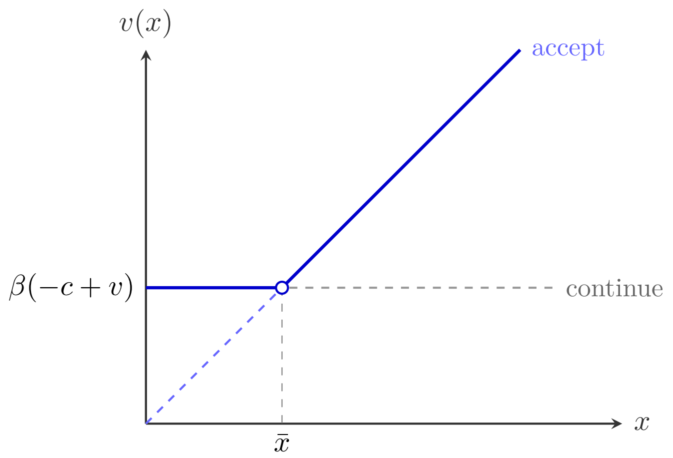

Figure 1: The horizontal dashed line represents the continuation value. The dashed 45-degree line is the value of the offer at hand. The solid line traces the maximum: decline offers below $\bar{x}$, accept offers above the reservation price.

[2]The problem is **recursive**: $v(x)$ is defined in terms of $v(x)$. Existence and uniqueness follow from a contraction-mapping argument under standard regularity conditions on F; see Lippman and McCall [1986]. We proceed directly to the solution.

In particular,

$$v(x) = \begin{cases} \bar{x} & \text{if } x < \bar{x} \\ x & \text{otherwise} \end{cases}$$

The probability an offer x falls below $\bar{x}$ is $F(\bar{x})$. We rewrite equation (3) as

$$v = F(\bar{x})\,\bar{x} + \int_{x \geq \bar{x}} x dF(x)$$

and substitute into the definition of $\bar{x}$:

$$\bar{x} = \beta\left(-c + F(\bar{x})\,\bar{x} + \int_{x \geq \bar{x}} x dF(x)\right)$$

Subtracting $\beta\bar{x}$ from both sides yields

$$\bar{x}(1-\beta) = \beta\left(-c + \int_{x \geq \bar{x}} (x - \bar{x})\, dF(x)\right)$$

because $\beta\bar{x} = \beta\bar{x}\left[F(\bar{x}) + 1 - F(\bar{x})\right] = \beta F(\bar{x})\,\bar{x} + \beta \int_{x \geq \bar{x}} \bar{x}\, dF(x)$. Rearranging produces the key optimality condition:

$$\bar{x} + \frac{\beta}{1-\beta} c = \frac{\beta}{1-\beta} \int_{x \geq \bar{x}} (x - \bar{x})\, dF(x) \tag{4}$$

The left-hand side is the cost of forgoing the offer $\bar{x}$; the right-hand side is the expected benefit from one more search.

1.2.4 A numerical example

Let F be uniform on $[0,1]$, so that $dF(x) = dx$ and offers are equally likely anywhere between 0 and 1. Set $c = 0.05$ and $\beta = 0.95$. The optimality condition (4) becomes

$$\bar{x} + \frac{0.95}{0.05} \cdot 0.05 = \frac{0.95}{0.05} \int_{\bar{x}}^{1} (x - \bar{x})\, dx$$

The integral evaluates to $\frac{(1-\bar{x})^2}{2}$, so

$$\bar{x} + 0.95 = 19 \cdot \frac{(1-\bar{x})^2}{2}$$

Solving numerically yields $\bar{x} \approx 0.60$. The probability of selling in any given period is $p = 1 - F(0.60) = 0.40$, and the expected time to sell is $\mathbb{E}[\tau^*] = 1/0.40 \approx 2.5$ periods.

If the seller becomes more impatient ($\beta = 0.80$), the same calculation yields $\bar{x} \approx 0.44$, so $p = 0.56$ and $\mathbb{E}[\tau^*] \approx 1.8$ periods. The impatient seller accepts lower offers, sells faster, and the asset becomes more liquid—consistent with the comparative static derived below.

Finally, suppose offers arrive only half the time ($\alpha = 0.5$) while $\beta = 0.95$ and $c = 0.05$. The reservation price drops to $\bar{x} \approx 0.46$ (the seller is less selective because opportunities are scarce), and $p_\alpha = 0.5 \times (1 - 0.46) = 0.27$, so $\mathbb{E}[\tau^*] \approx 3.7$ periods. The asset is less liquid despite the lower reservation price—here the direct effect of fewer offers dominates.

1.3 Liquidity and timeliness

We now map a definition given by Keynes [1936, p. 67] into our framework. Table 1 draws the connection.

According to Keynes	Our interpretation
at short notice	in one period
more certainly realizable	has a higher probability
without loss	in accord with the optimal policy

Table 1: Transferring Keynes' definition into a search framework.

Given that an offer with distribution F is drawn in every round, the probability the offer exceeds the threshold $\bar{x}$ is

$$p = 1 - F(\bar{x})$$

which decreases in $\bar{x}$. A **higher reservation price lowers the probability of selling the asset** in any given period.

The probability the asset is sold after exactly k rounds is

$$(1 - p)^{k-1} p$$

The first $k - 1$ offers were insufficient (below $\bar{x}$), but the kth draw exceeds the threshold. This describes a geometric distribution with mean

$$\mathbb{E}[\tau^*] = \frac{1}{p} = \frac{1}{1 - F(\bar{x})}$$

A **higher reservation price increases the holding time until the asset is sold, lowering its liquidity**.

1.4 Liquidity and time preferences

We reformulate the optimality condition (4) as

$$G(\bar{x}(\beta), \beta) = 0 = \frac{1 - \beta}{\beta}\bar{x} + c - \int_{x \geq \bar{x}} (x - \bar{x})\, dF(x)$$

and apply the implicit function theorem.[3] The partial derivatives are

$$\frac{\partial G}{\partial \beta}=-\frac{\bar{x}}{\beta^2},\qquad \frac{\partial G}{\partial \bar{x}}=\frac{1-\beta}{\beta}+1-F\left(\bar{x}\right)$$

so that

$$\frac{\partial \bar{x}}{\partial \beta}=-\frac{\partial G/\partial \beta}{\partial G/\partial \bar{x}}=\frac{\bar{x}/\beta^2}{\frac{1-\beta}{\beta}+1-F\left(\bar{x}\right)}=\frac{\bar{x}}{\beta-\beta^2F\left(\bar{x}\right)}>0$$

The **reservation price increases as the seller becomes more patient**. Therefore, **more impatient sellers raise liquidity** while patient sellers lower it.

1.5 Liquidity and market thickness

So far we assumed that offers arrive in every period. Extending the framework to address market thickness is straightforward. Suppose we receive an offer with probability α only. We extend the valuation (2) for the case when no offer arrives. An offer at hand is worth

$$v_\alpha\left(x\right)=\max\left\{x,\beta\left(-c+v_\alpha\right)\right\}$$

where v_α is the continuation value. This happens with probability α, and the offer is still drawn from F. If no offer arrives, we move forward:

$$\overline{v_\alpha}=\beta\left(-c+v_\alpha\right)$$

We extend (3) to

$$\begin{aligned} v_\alpha &= \alpha\int v_\alpha\left(x\right)dF\left(x\right)+\left(1-\alpha\right)\overline{v_\alpha}\\ &= \alpha\int v_\alpha\left(x\right)dF\left(x\right)+\left(1-\alpha\right)\beta\left(-c+v_\alpha\right)\\ \Leftrightarrow \left(1-\left(1-\alpha\right)\beta\right)v_\alpha &= \alpha\int v_\alpha\left(x\right)dF\left(x\right)-\left(1-\alpha\right)\beta c\\ \Leftrightarrow v_\alpha &= \frac{\alpha}{1-\left(1-\alpha\right)\beta}\int v_\alpha\left(x\right)dF\left(x\right)-\frac{1-\alpha}{1-\left(1-\alpha\right)\beta}\beta c \end{aligned}$$

Using the reservation value $\bar{x}_\alpha$ we have

$$v_\alpha=\frac{\alpha}{1-\left(1-\alpha\right)\beta}\left(\bar{x}_\alpha F\left(\bar{x}_\alpha\right)+\int_{x\geq\bar{x}_\alpha}xdF\left(x\right)\right)-\frac{1-\alpha}{1-\left(1-\alpha\right)\beta}\beta c$$

[3] Apply the Leibniz integration rule.

Since the continuation value equals the reservation value,

$$\bar{x}_\alpha = \beta \left(-c + \underbrace{\frac{\alpha}{1-(1-\alpha)\beta}\left(\bar{x}_\alpha F(\bar{x}_\alpha) + \int_{x\geq \bar{x}_\alpha} x dF(x)\right) - \frac{1-\alpha}{1-(1-\alpha)\beta}\beta c}_{v_\alpha} \right)$$

Rearranging and subtracting $\alpha\beta\bar{x}_\alpha$ from both sides yields another optimality condition:

$$G_\alpha(\bar{x}_\alpha(\alpha), \alpha) = \frac{1-\beta}{\beta}\bar{x}_\alpha + c - \alpha \int_{x\geq \bar{x}_\alpha} (x - \bar{x}_\alpha)\, dF(x) = 0$$

The partial derivatives are

$$\frac{\partial G_\alpha}{\partial \alpha} = -\int_{x\geq \bar{x}_\alpha} (x - \bar{x}_\alpha)\, dF(x), \qquad \frac{\partial G_\alpha}{\partial \bar{x}_\alpha} = \frac{1-\beta}{\beta} + \alpha(1 - F(\bar{x}_\alpha))$$

so the implicit function theorem yields

$$\frac{\partial \bar{x}_\alpha}{\partial \alpha} = -\frac{\partial G_\alpha / \partial \alpha}{\partial G_\alpha / \partial \bar{x}_\alpha} = \frac{\int_{x\geq \bar{x}_\alpha} (x - \bar{x}_\alpha)\, dF(x)}{\frac{1-\beta}{\beta} + \alpha(1 - F(\bar{x}_\alpha))} > 0 \tag{5}$$

A **higher arrival rate increases the reservation price.**

What is the effect on the time to sell? The probability of selling in each round was $1 - F(\bar{x})$ conditional on an offer arriving. Now

$$p_\alpha = \alpha(1 - F(\bar{x}_\alpha))$$

so that

$$\mathbb{E}[\tau^*_\alpha] = \frac{1}{p_\alpha}$$

and

$$\frac{\partial p_\alpha}{\partial \alpha} = (1 - F(\bar{x}_\alpha)) - \alpha f(\bar{x}_\alpha) \frac{\partial \bar{x}_\alpha}{\partial \alpha}$$

which **we cannot sign**. The sign is ambiguous because a thicker market has two opposing effects: more offers arrive (raising the sale probability directly), but the seller becomes more selective (raising $\bar{x}_\alpha$ and lowering the acceptance probability). Equation (5) shows that $\frac{\partial \bar{x}_\alpha}{\partial \alpha}$ can be large when the expected gains from search (numerator) are high relative to the marginal cost of being more selective (denominator).

1.6 Conclusion

We began with vague appeals to liquidity—a "quality," a "scale," a "bundle of measurable properties"—that could not rank two assets. The search framework delivers what those definitions lacked: a precise, endogenous measure grounded in optimizing behavior. Liquidity is the expected time to sell, determined by the agent's reservation price, which in turn depends on holding costs, patience, and market thickness.

The model is partial equilibrium: offers arrive exogenously and are drawn from a given distribution. A general-equilibrium treatment would endogenize the offer distribution itself. Nonetheless, the framework already clarifies a key insight: whenever agents are indifferent between two assets, the one with a lower return must be more liquid.

Key takeaways.

- Liquidity is measured by the expected time to sell, $\mathbb{E}[\tau^*] = 1/(1 - F(\bar{x}))$.
- The reservation price $\bar{x}$ balances the cost of forgoing the current offer against the expected benefit of continued search.
- More patient sellers set higher reservation prices, lowering liquidity.
- Thicker markets (higher α) raise the reservation price, but the net effect on liquidity is ambiguous.
- A good definition is precise, measurable, and grounded in a framework. The search-theoretic definition of liquidity satisfies all three criteria.

1.7 Further reading

An alternative to the sequential search formulation has the seller *post* a price and wait for a buyer to arrive—the competitive search literature [Moen, 1997]. In that setting, the posted price affects both the terms of trade and the rate at which buyers arrive, generating a richer interaction between price and liquidity. For a survey, see Lippman and McCall [1986]. For the connection between liquidity and asset prices, Amihud and Mendelson [1986] show that the bid-ask spread—a market microstructure measure of illiquidity—is priced in equilibrium.

1.8 Exercises

1. Set $c = 0$ in equation (4). Show that the reservation price satisfies $\bar{x}(1 - \beta) = \beta \int_{x \geq \bar{x}} (x - \bar{x})\, dF(x)$. Does $\bar{x}$ depend on F? Compute $\bar{x}$ for F uniform on $[0, 1]$ with $\beta = 0.95$.

2. Let F be uniform on $[0, 1]$ with $\beta = 0.9$ and $c = 0.1$. Compute the reservation price $\bar{x}$ and the expected time to sell $\mathbb{E}[\tau^*]$.

3. Suppose holding costs increase from c to $c' > c$. Using equation (4), show that the reservation price falls. What happens to liquidity?

References

Yakov Amihud and Haim Mendelson. Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2):223–249, 1986.

Kenneth E Boulding. *Economic analysis, 3rd edition.* Harper and brothers Publishers, London, 1955.

Jack Hirshleifer. Liquidity, uncertainty, and the accumulation of assets. *CORE Discussion Paper*, 6810, 1968.

John Maynard Keynes. *General theory of employment, interest and money.* Atlantic Publishers & Dist, 1936.

Steven A Lippman and John J McCall. An operational measure of liquidity. *The American Economic Review*, 76(1):43–55, 1986.

Helen Makower and Jacob Marschak. Assets, prices and monetary theory. *Economica*, 5 (19):261–288, 1938.

Espen R. Moen. Competitive search equilibrium. *Journal of Political Economy*, 105(2): 385–411, 1997.